Readiness Check: Political Economy and Economic Development

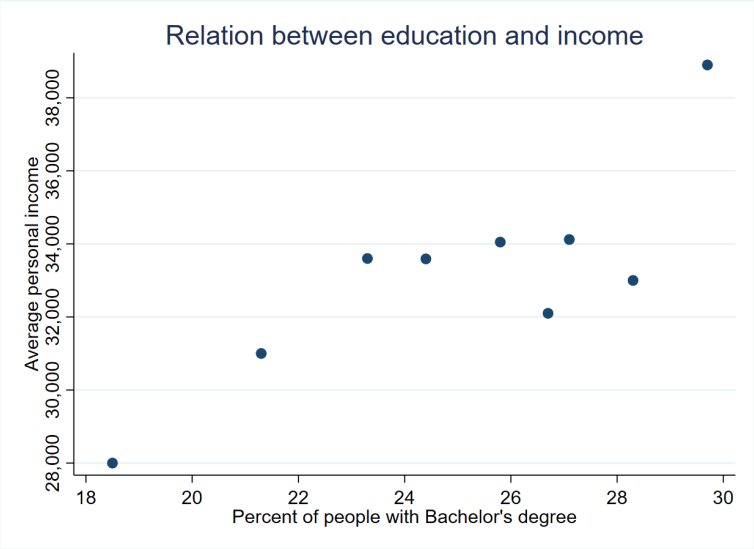
With Answers

While there are no prerequisites for this online course, it is recommended that learners have some familiarity with topics such as calculus, for example. Each question below is tied to concepts that will appear in this course, all of which it would be good to feel comfortable with. If you are new to these subjects, or eager to refresh your memory, please do consult the available resources provided on our website and within the courses, and be prepared to refer to these resources over the course of the class. Try to first answer these questions without consulting the resources, but fear not if you do consult them - being an agile user of outside resources will help you succeed in this course.

Useful Resources:

- *Basics of Regression:*
  - An Introduction to Linear Regression Analysis
  - Introduction to Regression Analysis: Causal Inference Bootcamp
- *Background on the development aid debate:*
  - Anti-aid: Bill Easterly: Why doesn't aid work?
  - Pro-aid: Jeffrey Sachs: The ethics and practicalities of foreign aid
  - RCTs/experimentation: Esther Duflo: Social experiments to fight poverty
- Fundamentals of calculus: Khan Academy: Calculus Tutorials
- Overview of advanced calculus: MIT Open Courseware: Multivariate Calculus
- Fundamentals of probability: Harvard University on edX: Fat Chance - Probability from the Ground Up

1. **Linear Regressions:** You are trying to determine the effect of education on income. The following graph plots data on the percent of adults with Bachelor's degrees in different states (x axis) and average income (y axis).



• How would a linear regression model adjusted to this data look in a graph? (.5 point)

Solution: A line. This is induced by a linear model by definition.

• If you used an OLS regression to fit a line to the data, would it be BLUE (best linear unbiased estimate)? (.5 point)

Solution: Yes. Linear regression is the best linear fit to the data in the sense that the linear model is the best linear predictor.

• What type of relationship do these variables seem to have? (.5 point)

Solution: There is positive correlation between percentage of adults with bachelors and mean earnings.

• Is this relationship causal? Why or why not? (.5 point)

Solution: No. Population may be different in different states for other reasons than education, that may induce variation in income too.

2. **Interpreting Regression Results:** The following table contains the results from regressing income on the percent of the population with a Bachelor's degree.

Parameter	**Estimate**	**Standard Error**	**P-value**
Intercept	14772	4086	0.0086
Slope	735	161	0.0026

• What does the intercept value tell us? Hint: recall the intercept is the point where a line crosses the y axis. (1 point)

Solution: Mean predicted earnings when the percent of adults with bachelor degrees is 0.

• What does the slope coefficient tell us? (1 point)

Solution: The increase in average income per additional percentage of adults with a bachelor degrees.

• Is the slope coefficient significant at the 10, 5, and 1 percent levels? Hint: recall that the p-value is the likelihood that a value of that size was obtained from a random distribution with mean 0. (1 point)

Solution: Based on the p-value, yes at 10, 5, and at 1. The p-value represents the likelihood that the observed result is induced by random variation in the sample. Thus, a lower p-value means that the results are more likely to be significant. The 10 percent significance level is obtained when the p-value is lower than 0.1, for 5 when the p-value is smaller than 0.05, and for 1 when the p-value is smaller than 0.01.

• Are we able to say what the effect of education on income would be for a higher percent of adults with Bachelor's degrees (e.g. more than 30 percent)? (1 point)

Solution: No. There may be other variables that are correlated with both education and income, and we have not collected data that would allow us to say anything definitively about the effect of education on income when over 30 percent of adults have Bachelor's degrees. As researchers, we might be able to formulate a hypothesis about this effect (especially after viewing this graph), but in order to say anything definitive we would need to collect and analyze more data.

3. **Marginal Cost and Benefit**: Assume you are trying to find the optimal quantity of medicines to have in a hospital with fixed capabilities (doctors) and fixed demand. You are interested in buying more medicine only if it is worthwhile in the sense that the acquired monetary benefit of each unit of medicine outweighs the additional cost, which is 10 USD.

   Assume the benefit is given by the following equation:

$$B = 20Q - .5Q^2$$

   where B = Benefits in USD and Q = Quantity.

   • What is the equation for the marginal benefit of medicine? (1 point)

   Solution: To find the marginal benefit, you take the derivative of the benefit equation. In this case, the answer is $MB = 20 - Q$.

   • What is the value of the marginal cost of medicine? (1 point)

   Solution: 10 because each unit of medicine costs 10 (marginal cost is 10).

   • What is the optimal quantity of medicine consumed? (1 point)

   Solution: The optimal value is found when MC=MB, so when $10 = 20 - Q$. This occurs when Q=10, so 10 is the optimal quantity consumed.

   • What would happen to the optimal quantity of medicine consumed if new technology reduces the medicine cost to 5? (1 point)

   Solution: Now the optimal value is found when MC=MB, so when $5 = 20 - Q$. This occurs when Q=5, so 5 is the optimal quantity consumed.

4. **Algebra:** Solve for x (1 point).

$$|-3x-4| = 5$$

Solution:

$$\begin{aligned} -3x-4 &= 5 \\ -3x &= 9 \\ x &= -3 \end{aligned}$$

$$\begin{aligned} -3x-4 &= -5 \\ -3x &= -1 \\ x &= \frac{1}{3} \end{aligned}$$

$$x = \Big\{-3, \frac{1}{3}\Big\}$$

5. **Integrals:** Compute the following integral (2.5 points).

$$\int_0^3 \int_0^2 xy^2 dx dy$$

Solution:

$$\begin{aligned} \int_0^3 \int_0^2 xy^2 dx dy &= \int_0^3 \Big(\frac{x^2}{2}y^2\Big|_{x=0}^{x=2}\Big) dy \\ &= \int_0^3 \Big(\Big[\frac{2^2}{2} - \frac{0^2}{2}\Big]y^2\Big) dy \\ &= \int_0^3 2y^2 dy \\ &= \frac{2y^3}{3}\Big|_{y=0}^{y=3} \\ &= \frac{2*3^3}{3} - \frac{2*0^3}{3} \\ &= 18 \end{aligned}$$

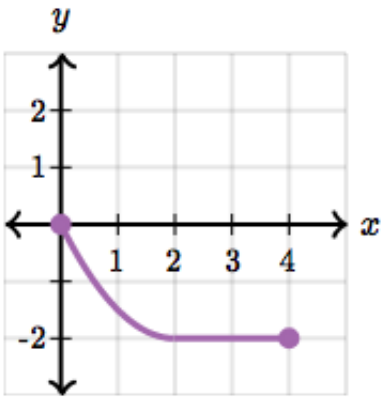
6. **Derivatives:** Compute the following derivative (2.5 points).

$$\frac{d}{dx}\Big[x^3 ln(x) + e^{3x}\Big]$$

Solution:

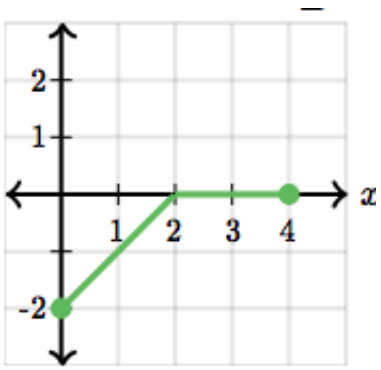
$$\begin{aligned} \frac{d}{dx}\Big[x^3 ln(x) + \Big] &= 3x^2 ln(x) + \frac{x^3}{x} + 3e^{3x} \\ &= 3x^2 ln(x) + x^2 + 3e^{3x} \end{aligned}$$

7. **Graphing Derivatives:** This is a graph of function $f(x)$:



What is the graph of its derivative, $f'(x)$?

Solution (2 points):



Note: *Question taken from Khan Academy.*

8. **Probability:** Two dice are rolled. Find the probability that the sum is equal to 4.

Solution (2 points):

The sample space consists of 36 outcomes: $\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),\ldots,(6,6)\}$. There are 3 outcomes that result in a sum of 4: $\{(1,3),(2,2),(3,1)\}$. Therefore, the probability that the sum of the faces is 4 is equal to $\frac{3}{36} = \frac{1}{12}$.

$$P(\text{sum of two dice is equal to 4}) = \frac{1}{12}$$